Bit Rate Determination for a Satellite Communications Link

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1.0 Abstract

This paper quantifies a way of maximizing a satellite link's bit rate once the transmitted Effective Isotropically Radiated Power (EIRP) of the satellite antenna is fixed. This maximization is done by fully utilizing the flexibility offered by some of the link variables. A practical scheme of achieving the maximum is discussed, and the result is compared with the theoretical maximum.

2.0 Introduction

Generally, the power available to every satellite subsystem is at a premium. A link design is usually done to optimize the power available to the telecommunications subsystem (Radio Frequency Subsystem, RFS) and to determine the bit rate to be used to transmit the data at reasonable data and carrier margins.

The satellite EIRP is fixed when the satellite transmitting antenna, the signal frequency, and the RF power radiated by the antenna are fixed. Generally, using the satellite EIRP, a bit rate is computed for acceptable link margins in the satellite link budget. The largest loss a link usually suffers is the propagation loss or space loss. The space loss goes through a cyclical variation for the Low Earth Orbiter (LEO) links; it is nearly constant for the Geostationary Earth Orbiter (GEO); and the rate of change is little for the Deep Space (DS) satellite links. For the circular orbits common to LEO satellites, the 10ss is largest when the satellite first becomes visible to the ground-station receiving antenna. The loss is least when the satellite is at the highest elevation angle to the receiving antenna and then reaches another peak when the satellite disappears over the horizon, as viewed from the ground station. This phenomena occurs because the satellite is closest to the receiving, antenna when its orbit brings it to the maximum elevation angle of the receiving antenna. It should be noted that this paper assumes an omni directional antenna on the satellite.

As the elevation angle to the receiving antenna increases, the propagation path through the atmosphere shortens and atmospheric losses correspondingly decreases. This is true for DS and LEO satellite links alike. Although this gain is not very much, it should be exploited, especially if the link margins are not particularly high. The third link variable that can be utilized to optimize the data rate selection is the receiver system temperature variation, which goes hand in hand with atmospheric losses. These effects are described Mow along with a method that utilizes the available power to increase the data rate, where possible. The variable data rate becomes necessary to maintain cost control of the satellite project ,

3.0 Theory

Although this theory can be developed for circular, elliptical, or DS (interplanetary) orbits, without loss of generality, we will use the circular orbit for illustration. Figure 1 shows the geometry of a circular orbit. The ground station will be assumed to be on the equator and the satellite will be assumed to be in the equatorial plane. These assumptions are made only to make the computations tractable and they are not absolutely essential for the conclusions that follow.

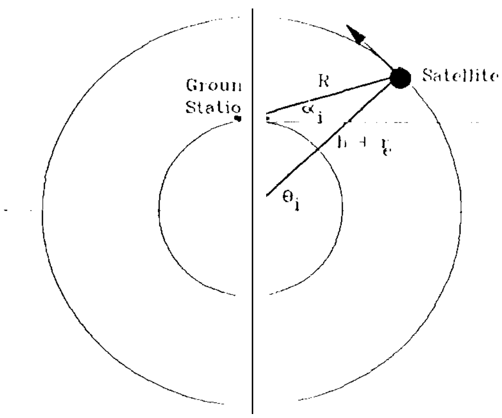

Figure 1, Geometry of the satellite orbit,

The ground station-to-satellite instantaneous range is a function of the satellite's position relative to the receiving ground station antenna and the Earth's rotation. From Figure 1 it is obvious that the range is greatest when the receiving antenna elevation angle is at a minimum. It should be noted that the practical minimum elevation angle at which satellite communications can be established depends upon the landmask of the ground station antenna site. As an example, the National Aeronautics and Space Administration (NASA)/Jet Propulsion Laboratory (JPL) tracking station located at the Goldstone complex has a landmask of about 6 degrees. in Figure 1, α is the elevation angle, h is the altitude of the satellite, r_e is the Earth's radius (Earth is assumed to be completely spherical with a radius = 6378 km), and R is the range. Using plane geometry, wc can easily calculate the range using the following equation:

Range at antenna elevation angle α, $R(\alpha)$

$$R(\alpha) = - r_e \operatorname{Sin}(\alpha) + \sqrt{r_e^2 \operatorname{Sin}^2(\alpha) + h(h + 2r_e)}$$

$$\text{With } \alpha = \alpha_{min}, \quad \text{max range} \triangleq R(\alpha_{min})$$

$$\triangleq R_{max}$$

$$\text{With } \alpha_{min} = 0, \ R(\alpha_{min}) = \sqrt{h(h+2r_e)},$$
$$\text{with } \alpha = 90 \text{ deg, zenith range} = h = \text{altitude} \qquad (1)$$

Figure 2 shows the satellite-to-ground station range as a function of the receiving antenna elevation angle. The propagation loss is a function of the range and frequency of the link and is given by

$$Propagation\,Loss \triangleq L_p = \left(\frac{c}{4\pi R f}\right)^2 \tag{2}$$

where R = R(α) is the range (km), f is the frequency (Hz), and c is the velocity of light (km/s).

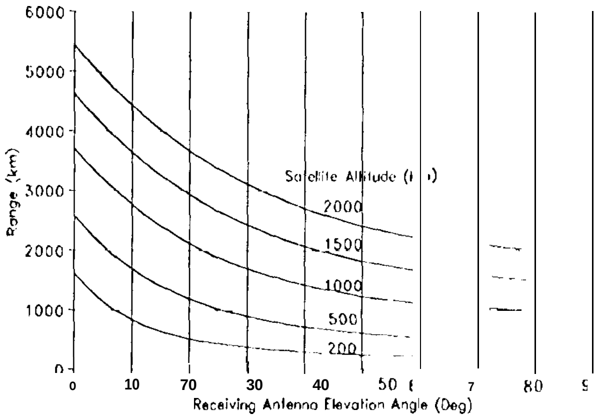

Figure 2. Variation of range as a function of the receiving antenna elevation angle.

Substituting the formula for range into equation (2), we obtain the propagation loss for the circular orbit as a function of the receiving antenna elevation angle. Since the range appears in the denominator, the propagation loss will be larger, (the ratio will become smaller) and the received power at the antenna terminals will be lesser. Thus the smallest propagation loss the link will experience is at the minimum range in the satellite orbit. Normalizing the loss by the maximum loss and converting the ratio into decibels, we obtain the following formula:

$$\frac{L_p}{L_{p\max}} = 20\,Log_{10}\left(\frac{R(\alpha_{min})}{R(\alpha)}\right) \quad (dB) \tag{3}$$

Although the normalized propagation loss varies considerably for LEO satellites, this normalized loss varies only slightly for DS satellites (for a day or small amounts of time) and is zero for the GEO satellites. GEO satellites will be excluded from further discussion because neither the satellite distance from the receiving station nor the antenna elevation angle changes appreciably. The range is a function of the satellite's position in its orbit and can be converted into the receiving ground-station antenna elevation angle, which may be converted into time. For the DS satellite, the percentage range may not change appreciably; however, the Earth's rotation produces a variation in the antenna elevation angle. Continuing the example of the equatorial circular orbit satellite with the receiving station on the equator, and assuming that the time count starts when the satellite first becomes visible to the ground station antenna, the time for any elevation angle, t_α, may be calculated using the angle subtended at the Earth's center as follows:

With $t_{\alpha_{min}} = 0$ sec,

$$t_\alpha = \frac{1}{2\pi}\left(\theta_\alpha - \theta_{\alpha_{min}}\right) T_{ap} \quad \text{Secs}$$

Where $\theta_\alpha = \cos^{-1}\left[\frac{R(\alpha)\cos(\alpha)}{r_c + h}\right]$ and

$$\theta_{\alpha_{min}} = \cos^{-1}\left[\frac{R(\alpha_{min})\cos(\alpha_{min})}{r_c + h}\right]$$

T_{ap} = Adjusted Orbital Period

$$= \frac{173860\,\pi\,(r_c + h)^{3/2}}{86400\,\sqrt{\mu}\ 2:2z(re\text{-}th)3'2} \quad \text{Sees} \tag{4}$$

where μ is Kepler's constant and has the value 398,613.52 km^3/s^2, a + sign is used for a retrograde satellite and a - sign for a prograde satellite.

in the case of a DS satellite, if the satellite is in the orbital plane of the planets, and time count starts at the moment the satellite becomes visible to the ground station, then time and elevation angle of the receiving antenna can be described approximately by Eq (4), but with 43200 replacing T_{ap}. The same formula can also be used for an elevation angle of 90 to 180 degrees, by having α = elevation angle -90. It should be noted that for both LEO and DS orbits, if the latitude of the ground station is not O degrees or the orbit is not in the equatorial plane, a more involved effort will be needed to obtain time as a function of the receiving antenna elevation angle.

The major contributing factors to system noise temperature are weather conditions around the receiving antenna, the frequency used by the link, temperature of the antenna itself, receiver system components such as the Low Noise Amplifier (LNA), and the receiver noise figure. After taking measurements of the noise temperatures at the receiving station for extended periods of time, we can produce a weather model for that particular station site, The weather model employed by NASA/JPL, for its tracking stations will be used in this paper. This model is described in greater detail in Ref. 1; however, it will be reproduced here in sufficient detail to be useful for our purposes. The model uses the measured values of atmosphere noise temperature and attenuation at zenith as a function of weather condition or cumulative weather distribution, (CD) around the antenna. Using the tabulated data, the mean physical temperature of the atmosphere is modeled as

$$T_p = 265 + 15\ CD \quad (K) \tag{5}$$

It should be noted that the maximum value of the physical temperature is 280 (K). This clearly shows the subjective nature of the model in terms of the station site. The atmospheric attenuation is a function of the antenna elevation angle and is given by the following equation:

$$A(\alpha) = \frac{A_{zen}}{\sin(\alpha)} \quad (dB) \tag{6}$$

where A_{zen} is the zenith atmospheric attenuation measured in dB and α is the antenna elevation angle. Given this information, the noise temperature due to the atmosphere, T_{atm}, can be calculated by using the following formula:

$$T_{atm}(\alpha) = T_p\left[1 - 10^{-A(\alpha)/10}\right] \quad (K) \tag{7}$$

It should be noted that T_{atm} is the temperature increase and should be added to the system noise temperature to get the total system noise temperature. Similarly, the cosmic background noise temperature contribution (Ref. 1) can be computed as

$$T_c(\alpha) = C\,10^{-A(\alpha)/10} \quad (K) \tag{8}$$

The factor C has a value ranging from 2 to 2.7 depending upon the frequency used for the link. Cosmic background noise is not appreciable (generally below 5 K) and hence will be dropped from further consideration. The atmospheric attenuation and system temperature increase are functions of the antenna elevation angle, and each of these effects reaches a maximum when the elevation angle is a minimum, usually the landmask angle of the antenna.

Dividing each of the effects by their respective maximum, converting them to dB, and adding them to Eq. (5) produces Eq. (9) which shows the variation of the link losses according to changing elevation angles. As the elevation angle increases from horizon to zenith, the power attenuation experienced by the link decreases to a minimum. Since the satellite radiates a constant EIRP, as the losses decrease, extra power becomes available for the link. This additional power results in a better performance or decreased bit error rate at the receiver. Keeping the bit-error rate and the link margins at a desired level, as the antenna elevation angle increases, the freed power may be used to increase the transmitted data rate. Thus Eq. (9) precisely equals the ratio (in dB) of the bit rate of the link at a given elevation angle and the bit rate at the lowest possible elevation angle (generally the landmask angle) at which the link is still feasible.

$$\Delta\text{Gain} = \begin{matrix}\text{Bit Rate}\\ \text{Ratio}\end{matrix} = 10\,\mathrm{Log}_{10}\left[\frac{T_{syst} + T_{atm}(\alpha_{min})}{T_{syst} + T_{atm}(\alpha)}\right] + \left(\frac{A_{zenith}}{\mathrm{Sin}(\alpha_{min})} - \frac{A_{zenith}}{\mathrm{Sin}(\alpha)}\right) + 20\,\mathrm{Log}_{10}\left[\frac{R(\alpha_{min})}{R(\alpha)}\right] \quad (dB) \tag{9}$$

Figure 3 plots the bit-rate ratio defined in Eq. (9) in dB versus the elevation angle (using the previous convention for elevation angle). The parameter of the graph is the altitude of the satellite which has values 200, 500, 1000, 1 500, 2000 km. and α_{min} is assumed to be 6 degrees. In every case, when the antenna is pointed to the zenith i.e., at 90 degrees elevation angle, the bit-sate ratio peaks because losses are at a minimum at this elevation angle. For example, at a satellite altitude of 1000 km, the peak of normalized bit-rate is about 12 dB. This implies that, with the same link margins, the bit rate at the zenith could be about fifteen times the bit rate at the landmask angle. The bit rate ratio becomes more peaked as the altitude of the satellite decreases.

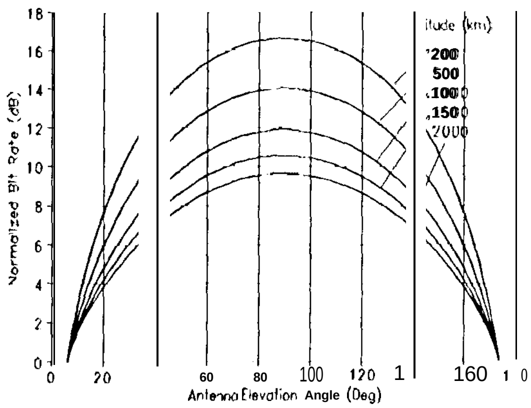

Figure 3. Normalized bit rate versus the receiving antenna elevation angle for LEO satellites.

Using Eq. (4), we can redraw Figure 3 with the bit-rate ratio as a function of time, The time count starts when the satellite first becomes visible to the receiving station antenna at the landmask angle and stops when the satellite disappears over the horizon.

Figure 4 shows these curves. It should be noted that the x axis is time, the y axis is not in dB, and The visibility time changes for different satellite altitudes. However, the antenna elevation angle can only vary from the landmask angle to the zenith or 90 degrees, which is same for any altitude of the satellite.

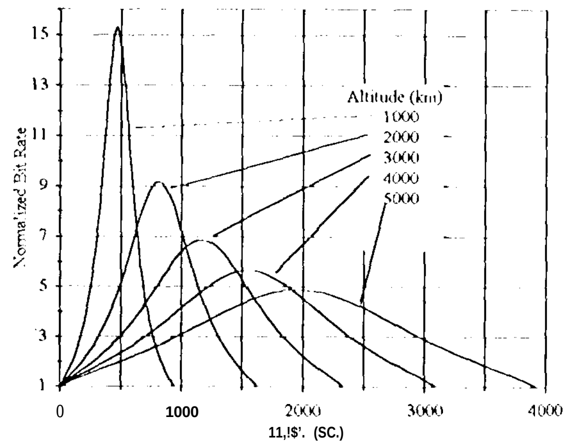

Figure 4. Normalized bit rate versus time.

Figure 5 draws Eq. (9) minus the space loss term versus the elevation angle, with the landmask angle still at 6 degrees. It shows the bit-rate increase possible for a DS satellite as the antenna elevation angle varies from landmask angle to the zenith and back. While the range does not change at all (or much) with the elevation angle, the allowable bit-rate increases with system noise temperature and atmospheric effects.

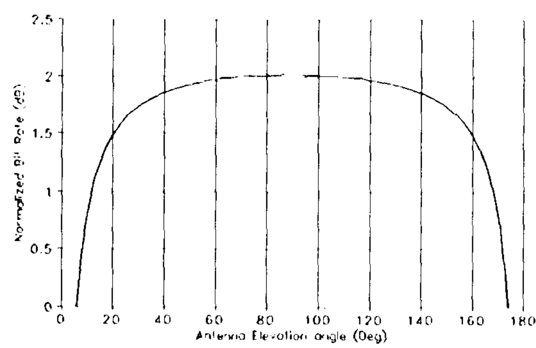

Figure 5. Normalized bit rate versus receiving antenna elevation angle for a DS satellite.

4.0 Communications Link Bit-Rate and Bit-Transmission Efficiency

The previous equations show that losses encountered by the link are reduced as the ground receiving antenna's elevation angle increases from horizon to zenith. This gain in power may be used by the link to improve the bit-rate. Figure 4 shows the increased bit-rate normalized by the lowest supportable bit rate at the desired link margins. To fully use the gain advantage, the satellite must have a continuously variable bit rate, as shown by curves of Figure 4. Since it may be difficult to achieve a continuously variable data rate in the satellite circuitry, we can approximate the variable bit rate by a stepped approximation of the curve i.e., use a few switchable bit rates. Figure 6 shows four satellite bit rates at an altitude of 1000 km. It should be noted that the area under the curve multiplied by the lowest bit rate at the landmask angle at the desired link margins provides the total number of bits received in a single pass of the satellite.

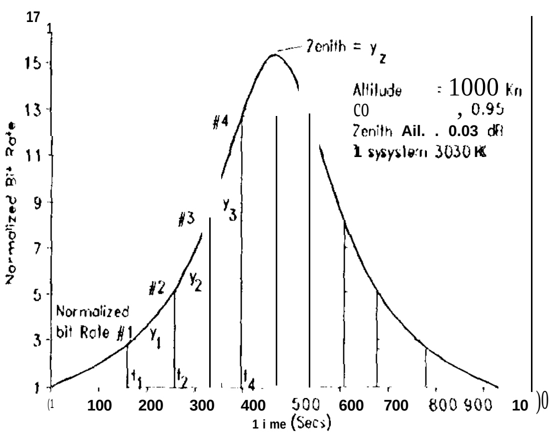

Figure 6. Normalized bit rate versus time with the switchable bit rates

The figure demonstrates that the area under the switchable bit rates is smaller than that under the continuous curve, this leads to the following definition of bit rate efficiency

$$\text{Bit Transmission Efficiency} : \eta$$

$$= \frac{\text{Total Number of Bits Received by the Ground Station}}{\text{Maximum Number of Bits Possible Over the Link}} \qquad (10)$$

Using a normalization factor equal to the minimum bit rate possible for the link at the landmask angle, the definition of the bit-transmission efficiency can be modified to

$$\text{Bit Transmission Efficiency} = \eta$$

$$= \frac{\text{Normalized Total Number of Bits Received by the Ground Station}}{\text{Normalized Maximum Number of Bits Possible Over the Link}} \qquad (11)$$

If f(t) is the function defined by Eq. (9) converted to a number from dB, the denominator of the above equation can be calculated as:

$$\text{Normalized Maximum \# of Bits} = \int_0^{t_{zenith}} f(t)\,dt \qquad (12)$$

Figure 7 plots Eq. (12) and the visibility time as a function of the satellite altitude To select switchable bit rates, one may select an equal amount of time for each of the bit rate (a fixed number) to remain active and then compute the efficiency.

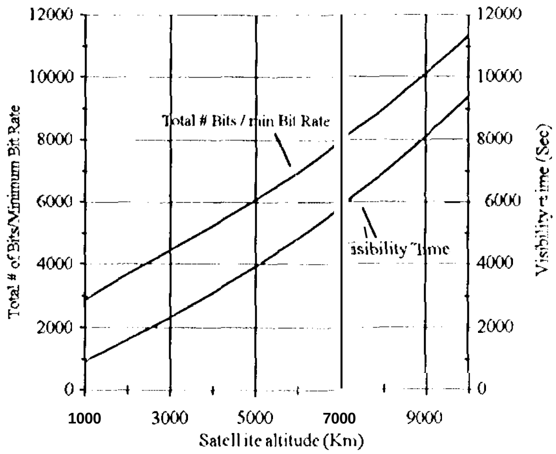

Figure 7, Normalized total number of bits and visibility of satellite versus satellite altitude.

This procedure may not yield optimal arrangement in terms of the bit-rate efficiency when the number of switchable bit rates is small, but it gives reasonably good efficiency for larger numbers of bit rates. It should be noted that the equal time for each bit rate does not necessarily mean an equal traverse of the receiving antenna.

Assuming that there are n bit rates available to the satellite transmitter/transponder and that the bit rates will remain active for equal amounts of time, $\Delta t = t_{j+1} - t_j$, the bit-transmission efficiency can be given as

$$\eta = \frac{\Delta t \left(1 + \sum_{i=1}^{n} y_i\right)}{\int_0^{t_{zenith}} f(t)\, dt} \Rightarrow \operatorname{Limit}_{n \to \infty} (\eta) = 1 \qquad (13)$$

Factor y_i, which is not in dB, is defined in Figure 6. With a large enough number of bit rates that allows the trapezoidal rule to be used to generate the area under the curve, the efficiency formula becomes

$$\eta = \frac{1 + \sum_{i=1}^{n} y_i}{1 + 2\sum_{i=1}^{n} y_i + y_Z} \qquad (14)$$

Factor y, can be calculated (y_Z is the value of y at the zenith) using the following procedure:

$$\Delta t = -\frac{nap}{2\pi(n+1)} \times$$

$$\left| \frac{\pi}{2} - \operatorname{Cos}^{-1}\left(\frac{R(\alpha_{min}) \operatorname{Cos}(\alpha_{min})}{r_e + h} \right) \right|;$$

The variable α_{min} was defined earlier

$$\theta_i = 2\pi\left(1 - \frac{i\Delta t}{T_{ap}}\right) +$$

$$\operatorname{Cos}^{-1}\left(\frac{R(\alpha_{min}) \operatorname{Cos}(\alpha_{min})}{r_e + h} \right);$$

$$\alpha_i = \operatorname{Tan}^{-1} \left| \frac{\left(1 + \frac{h}{r_e}\right) \operatorname{Sin}(\theta_i) - 1}{\left(1 + \frac{h}{r_e}\right) \operatorname{Cos}(\theta_i)} \right|$$

$$y_j = 10\,\mathrm{Log}_{10}\left(\frac{T_{syst} + T_{atm}(\alpha_{min})}{T_{syst} + T_{atm}(\alpha_j)}\right) + \frac{A_{zen}}{\mathrm{Sin}(\alpha_{min})} - \frac{A_{zen}}{\mathrm{Sin}(\alpha_j)} + 20\,\mathrm{Log}_{10}\left(\frac{R(\alpha_{min})}{R(\alpha_j)}\right) \tag{15}$$

Figure 8 plots Eq. (11). in this figure, the number of bit rates on the x axis is in addition to the bit rate feasible when the satellite first becomes visible to the receiving antenna. in actual practice, the number of switchable bit rates will not be the same for the rising and setting of the satellite, depending upon the ellipticity of the orbit, the orbital inclination, and the ground-station latitude. However, the procedure of finding the bit-rate efficiency essentially remains the same.

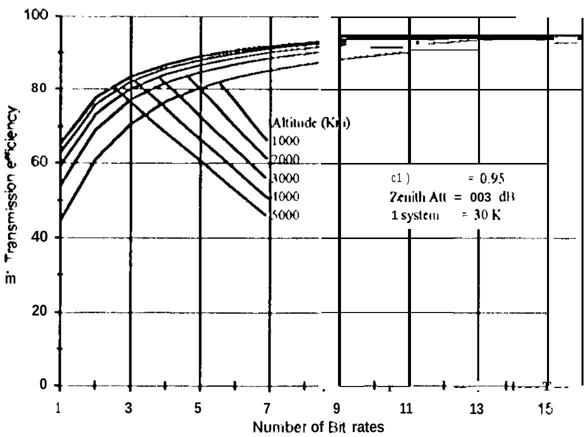

Figure 8. Bit transmission efficiency versus the number of bit rates.

5.0 Conclusions

The factors affecting the determination of a satellite link bit rate were presented. Ideally, to realize the 100 percent bit transmission efficiency one needs a continuously variable bit rate, the paper presented the necessary variation of the bit rate. Figure 6 shows that for a satellite at an altitude of 1000 km, can have a over 80°A bit-transmission efficiency with five or more switchable bit rates. The same procedure can be applied to compute the number of switchable bit rates necessary for any given altitude of the satellite.

REFERENCES

1. "Atmospheric Attenuation and Noise Temperature," Deep Space Network/Flight Project Interface Design Handbook, 11810-5, Rev. D, Volume 1, TCI-40, Rev. C, pages 3-7.